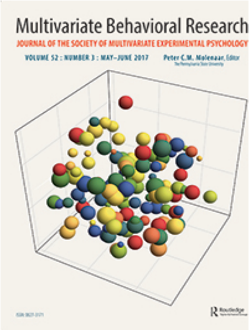

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Friendship Dissolution Within Social Networks Modeled Through Multilevel Event History Analysis

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ABSTRACT

A social network perspective can bring important insight into the processes that shape human behavior. Longitudinal social network data, measuring relations between individuals over time, has become increasingly common—as have the methods available to analyze such data. A friendship duration model utilizing discrete-time multilevel survival analysis with a multiple membership random effect structure is developed and applied here to study the processes leading to undirected friendship dissolution within a larger social network. While the modeling framework is introduced in terms of understanding friendship dissolution, it can be used to understand microlevel dynamics of a social network more generally. These models can be fit with standard generalized linear mixed-model software, after transforming the data to a pair-period data set. An empirical example highlights how the model can be applied to understand the processes leading to friendship dissolution between high school students, and a simulation study is used to test the use of the modeling framework under representative conditions that would be found in social network data. Advantages of the modeling framework are highlighted, and potential limitations and future directions are discussed.

KEYWORDS

Event history analysis; generalized linear modeling; multilevel modeling; social network analysis; survival analysis

In recent years, psychologists and other social scientists have become increasingly interested in analyzing network data, recognizing that social networks play a key role in people's lives (Borgatti, Mehra, Brass, & Labianca, 2009; Wasserman & Faust, 1994; Snijders, 2005a). For example, attributes of adolescent peer networks are important predictors of an individual's substance use (Ennett et al., 2006), and socialization and social selection within networks play important roles in shaping adolescents' religious beliefs and behaviors (Cheadle & Schwadel, 2012). The rapid rise and influence of virtual social networks such as Facebook and Twitter have also highlighted the existence of network structures and have become a focus of research as well as a vehicle for studying network effects for many researchers (Cha, Haddadi, Benevenuto, & Gummadi, 2010; Romero & Kleinberg, 2010).

Friendships are naturally embedded within social networks. As time passes and circumstances or relationships change, friendships can dissolve—often with "considerable distress" (Baumeister & Leary, 1995, p. 503). Researchers have found that friendship serves different purposes for men and woman; from as early as preschool to adulthood, gender differences within friendships have been noted (Maccoby, 1990; Johnson & Aries, 1983). Females have been found to develop closer and more intimate relationships in general, and some researchers argue dissolution tends to be more significant for females as a result (Jalma, 2008). While dissolution is an important phase of many relationships, most research has focused solely on the mechanisms leading to friendship formation. Schaefer, Kornienko, and Fox (2011), for example, found that depression homophily could be created in a network through a withdrawal mechanism even in the absence of preference. However, comparatively little research has analyzed the factors leading to friendship dissolution, and the research that has focused on dissolution has largely focused on romantic relationships (Sprecher, 1988; Felmlee, Sprecher, & Bassin, 1990). A notable exception is a recent study of adolescent friendship dissolution that investigated whether these relationships dissolve because of differences between the individuals, characteristics of the friends, or both (Hartl, Laursen, & Cillessen, 2015).

Although a variety of models have been developed to identify and study the basic structure of social networks, many fundamental questions surrounding friendship dissolution remain stubbornly difficult to address using current analysis approaches. Example questions include "What are the processes leading to a friendship ending and what's the role of the individuals' depression in this

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process?" as well as "How long does a typical friendship last?" and "Does friendship duration differ between girls and boys accounting for other factors?" The aim of these questions is to understand not simply friendship length, but also how different covariates affect dissolution, controlling for other covariates. One common aspect of these questions is that they share a concern with dyadic bonds within a network (e.g., specific friendships) rather than a concern with the global network structure. Another is that they focus on understanding the processes leading to—as well as timing of—an event occurring between two individuals. The aim of this article is to provide a modeling framework for addressing such questions, accounting for the fact individuals are members of multiple relationships within a larger social network.

In this article, extant modeling approaches for addressing social network questions are first briefly introduced. To better address the example questions, these methods are then built upon in order to develop a general modeling framework that utilizes event history analysis accounting for the special features (e.g., dependencies) present within network data with a multiple-membership random-effects structure. This modeling framework is tested through a simulation study under representative conditions that would be found in social network data and then demonstrated through an empirical application to high school social network data.

Friendship duration model

Building upon a prior application of multilevel event history analysis to social network data by de Nooy (2011), the "friendship duration" (FD) model is developed here to study the processes leading to the occurrence and timing of friendship dissolution between individuals embedded within a network. While building on prior work, the model in this article is to our knowledge the first to apply multilevel event history analysis to an undirected network and, relatedly, the first to utilize a multiple membership random effect structure (necessary due to the undirected nature of the network) as described in more detail in the following. This model development was motivated by an aim to model the processes leading to high school friendship dissolutions, specifically to understand the role of gender and depression in friendship dissolution, which is examined in the empirical example later in this article.

Statistical models with a social network perspective are often concerned with modeling the processes leading to a network's structure, known as social-selection processes (Daraganova & Robins, 2013; Anderson, Wasserman, & Crouch, 1999). In other words, the entire network is often the outcome of interest. For example, network structure—specifically, the formation of ties between nodes, such as friendship nominations between peers—can be modeled using exponential random graph models (ERGMs), commonly referred to as p^* models (Wasserman & Robins, 2005; Frank & Strauss, 1986). ERGMs are built on the idea that social networks are stochastic and that the patterns evident within a given network can be seen as evidence of specific local processes, such as reciprocity, transitivity, and homophily[1] (Wasserman & Pattison, 1996).

Longitudinal social network models have become increasingly popular in recent years to understand network evolution. Many of these models are based on the assumption that observed networks are the result of a continuous-time Markov process (Snijders, 2005a). Longitudinal network data generally enable researchers to better understand the dynamics leading to an observed network at a particular point in time as one can condition on the prior configuration of the network to better understand the subsequent changes. For example, the actor-oriented model as implemented in the software program SIENA is a general and flexible framework that allows the probabilities of relational changes to depend on the entire network structure, with actors assumed to change their ties to optimize an objective function (Snijders, 2005a). SIENA has been used to study selection, dissolution, and socialization processes of happiness in adolescent friendship networks, for example (van Workum, Scholte, Giletta, Cillessen, & Lodder, 2013). Alternatively, Hanneke, Fu, and Xing (2010) build directly on cross-sectional ERGMs to model longitudinal network data with a model referred to as a temporal ERGM (TERGM), adding an exponential family function for the transition probability of a network from one period to the next.

While these models provide a framework from which to evaluate the processes leading to the network's structure, they are not structured to understand whether and when a specific type of event occurs between individuals embedded within the network. They are thus also limited with respect to answering questions regarding the processes leading to the event, such as those questions listed at the start of this article. In contrast to standard social-selection network models, the proposed modeling framework was developed to answer research questions about temporal characteristics of friendships—whether and when friendship dissolution occurs between individuals embedded within a network—and is not a general-purpose social-selection model. The proposed modeling framework does not aim to understand the processes leading to the entire network structure, but rather the processes leading to the occurrence and timing of friendship dissolution between specific dyads within a network using multilevel event history analysis.

[1] *Reciprocity* is the tendency for nodes to form directed relationships to alters who have initiated a directed relationship to the node. *Transitivity* is the tendency for a "friend of a friend" to become a friend. *Homophily* is the tendency of nodes to form relations with those who are similar to themselves.

Event history analysis

Event history analysis is at the core of the proposed framework as it is useful for understanding both whether and when events occur (event history models are also known as survival models, duration models, or failure-time models). In the social and behavioral sciences, the timing of an event can often be considered a discrete variable as data are often collected from a panel study or the year of an event is measured rather than the exact timing. This article thus focuses on discrete-time event history models, which can easily handle "tied" event times where two or more people have the same event time (Singer & Willett, 1993; Freedman, Thomton, Cambum, Alwin, & Young-DeMarco, 1988; Blossfeld, Hamerle, & Mayer, 1989). Discrete-time models can be used as an approximation to the results of a continuous-time survival analysis if continuous-time data were indeed available (Vermunt, 1997). Also important for the purposes here, discrete-time methods extend to incorporate time-varying covariates and a multilevel structure relatively easily (Steele, 2008; de Nooy, 2011).

To formalize discrete-time univariate event history analysis in terms of friendship dissolution, let us assume for now that friendship dissolution is nonrepeatable; that is, a dyad may only dissolve a friendship once.[2] For each period, we will define a binary outcome y, equal to 0 for periods during which the ties remain mutual and 1 for the period when the ties are no longer mutual. Let the random variable E denote the friendship dissolution event and t the discrete-time period, with $t = 1, 2, \ldots T$. It is likely that not all friendships will dissolve during the study, resulting in censoring and making the probability mass function f_t of E difficult to compute. However, a function known as the hazard can be utilized instead by building on the conditional nature of event occurrence. In this case, the hazard function is the conditional probability of friendship dissolution at a certain period, given the friendship had continued up to that period (Singer & Willett, 2003). The hazard is thus the unique risk of friendship dissolution at a given period, for those eligible to experience the event:

$$h_t = P(E = t|E \geq t) = P(E = t|E > t - 1) = \frac{P(E = t)}{P(E > t - 1)}. \quad (1)$$

The hazard of event occurrence is a conditional probability bounded between 0 and 1 and can be modeled with a generalized linear model (GLM) (Hedeker, 2005; Agresti, Booth, Hobert, & Caffo, 2000; Singer & Willett, 1993). With the unit of observation denoted i, a model with a vector of predictors $\boldsymbol{x}$ and associated vector of fixed effects $\boldsymbol{\beta}$ may be written as

$$\begin{aligned} logit\ (h_{it}) &= \boldsymbol{x}'_{it}\boldsymbol{\beta} \\ y_{it} &\sim Bernoulli\ (h_{it})\ . \end{aligned} \quad (2)$$

This model is similar to what was used by Hartl, Laursen, & Cillessen (2015) to model friendship dissolution of adolescents starting in seventh grade. However, an important limitation of this model is that it assumes that friend pairs are independent, an assumption that is not met when individuals are part of multiple relationships, as is the case when examining friendships within a social network. The effect of the violation of this assumption and approaches to account for the dependencies are discussed in more detail in the following.

While event history analysis is commonly used in medical research, relatively few applications exist of event history analysis to social network data and questions. Focusing on models for social-selection processes, applications are especially rare and nearly exclusively rely on continuous-time measurement of the network (Krempel, 1990; Hu, Kaza, & Chen, 2009; Kossinets & Watts, 2009; Butts, 2008). Others have focused on the propensity to be involved in ties but not specific tie formations (Robinson & Smith-Lovin, 2001; Tsai, 2000; Kim & Higgins, 2007). However, the majority of applications involving event history analysis and network data are social influence applications where network features are used as predictors in an event history analysis of another outcome, such as understanding the effect of the composition of a person's network on the person's health (Adams, Madhavan, & Simon, 2002; Villingshoj, Ross, Thomsen, & Johansen, 2006).

Event history models have rarely been used for social selection-processes because of the complexities arising from the dependencies in network data. Researchers applying event history models to longitudinal social network data have largely either ignored dependencies (Hartl, Laursen, & Cillessen, 2015; Krempel, 1990), sampled pairs from an extremely large network in order to obtain independent observations (Kossinets & Watts, 2009), or used a fixed-effects approach (by, for example, adding a dummy intercept in the model for all persons except one; Butts, 2008; Brandes, Lerner, & Snijders, 2009). None of these approaches is fully satisfying. Ignoring dependencies can result in biased parameter estimates and standard errors, potentially leading to incorrect conclusions (Guo & Zhao, 2000). Sampling pairs from an extremely large network, although effective, is obviously applicable only in limited circumstances. Last, while a fixed-effects approach does account for dependence, it is not parsimonious, does not allow for inference beyond the individuals within the sample, and makes it difficult to examine person-level predictors (Snijders, 2005b, p. 665).

[2] This assumption can be relaxed, as discussed in the Limitations and extensions section at the end of the article.

Fortunately, multilevel event history analysis can provide a parsimonious framework for accounting for dependencies inherent in social network data (Steele, 2011; Goldstein, 2011; Rabe-Hesketh & Skrondal, 2012). These models allow examination of individual and dyad-level covariates simultaneously, permit the evaluation of time-varying network context effects, and allow inferences to be made to the population from which the sample is drawn (Raudenbush & Bryk, 2002). Recently, De Nooy (2011) applied a multilevel event history model to model the dynamics of longitudinal network data on the timing of book reviews, utilizing cross-classified random effects to account for the individual tendencies of book authors and reviewers within the directed social network. Predictors within this model could include attributes of the book authors, the book reviewers, and the pair together (e.g., similarity of the two individuals on an attribute in order to test homophily). These predictors could also include characteristics of local network structure such as transitivity—the tendency for a "friend of a friend" to become a friend.[3] Discrete-time multilevel event history models are part of the generalized linear mixed model (GLMM) family and can thus be fit using software that fits GLMMs such as SAS, MLwiN, and R (Tuerlinckx, Rijmen, Verbeke, & De Boeck, 2006). Whereas de Nooy's model is appropriate for modeling directed social relationships, the goal of the current article is to propose a model that will account for the unique features of reciprocated or nondirected relationships, such as friendships.

Multilevel event history analysis

Building upon the event history model in Equation (2) but accounting for dependencies in social network data, the FD model is now developed in the following. For the purposes of this model, friendship can be defined to start when both individuals nominate each other as a friend in a directed network or a friendship is recorded for a pair in an undirected network.[4] Friendship dissolution occurs when the mutual nomination of friendship no longer occurs at a given period. More generally, this proposed model is useful for understanding the duration of reciprocated ties in a directed network or the duration of a tie in an undirected network. As the model is concerned with the duration of the dyadic relationship, the process being studied is dissolution rather than selection (e.g., examining whether and when the relationship ends rather than why the relationship began in the first place).

Right censoring is likely to occur in data measuring friendship dissolution events as some individuals will remain friends throughout the observation period, and the dissolution time—if it occurs—is thus unknown.[5] However, unlike most event history analyses, left censoring may also occur relatively often depending on how the network is sampled as individuals may be friends before observation begins.[6] If start times of the friendships are known, these left-censored event times may be incorporated by the conditional likelihood approach as outlined by Guo (1993). Thus, one potential solution, which must be considered before sampling data, is to ask participants to recall when the friendship first began in addition to asking about their current friendships. Alternatively, careful selection of the observation period may allow one to define the friendship for specific periods. For example, when sampling a friendship network from the beginning of high school to the end of high school, no left-censored data would exist if the event process under study is "how long do high school friendships last," as the event process is thus defined to begin at the start of high school at the earliest irrespective of whether individuals were friends prior to high school; this is the approach utilized for the empirical example in this article. Other alternatives include simply deleting left-censored observations (although this risks a potential selection bias) (Cain, et al., 2011) and incorporating the left-censored observations through more sophisticated techniques, such as imputing event times (Karvanen, Saarela, & Kuulasmaa, 2010), which are not discussed in this article.

The data set can be arranged in a "pair-period" structure where there is a row for each pair by time combination during which the pair is eligible to experience friendship dissolution. An example data set for the model is given in Table 1, where Friend 1 and Friend 2 are an arbitrary numbering of individuals within the pair. A time of 0 signifies the first period that a friendship is eligible to dissolve after relationship formation, and the outcome of

[3] Network context covariates can be calculated by counting subnetworks created by the formation (or dissolution) of the tie being modeled from links that precede this tie. The time ordering of the network thus allows network effects beyond the dyad to be included in the model in a straightforward manner without the "circular dependencies" that prohibit their inclusion in cross-sectional models of networks. One approach to including network context is to limit the context considered to lines appearing in the previous period through a retrospective sliding window approach. Alternatively, a decay function may be used to weigh network context by the length of time passed.

[4] When the relationship between two units (such as people in this case) is undirected, if there is a relationship between person A and person B, then there is assumed to be a relationship between person B and person A (e.g., Facebook network). By contract, a directed network might reveal a relationship from person A to person B but not from person B back to person A (e.g., Twitter network).

[5] Right censoring is when the event of interest does not occur during the timeframe of the study (in the friendship duration case, this occurs when a friendship does not end during the study period).

[6] Left censoring occurs when the event process begins before the observation period. Unlike right censoring, which is easily dealt with in the formulation of the likelihood, left censoring is not as straightforward to accommodate unless the hazard function is assumed to be constant over time (Guo, 1993). However, erroneously assuming a constant hazard can lead to severe bias in parameter estimates (Heckman & Singer, 1986).

Table 1. Construction of pair-period data set: Traditional network formulation at Wave 1.

Person	1	2	3	4
1	0			
2	0	0		
3	0	0	0	
4	0	1	0	0

Pair-period data set.

Pair	Friend 1	Friend 2	Time	Event	Wave
1,2	1	2	0	0	3
1,2	1	2	1	0	4
1,2	1	2	2	1	5
1,3	1	3	0	0	2
1,4	1	4	0	0	4
2,4	2	4	0	0	1
2,4	2	4	1	0	2
2,4	2	4	2	0	3
3,4	3	4	0	0	3
3,4	3	4	1	1	4
3,2	3	2	0	0	5
3,2	3	2	1	0	6

Note. A "1" represents a friendship exists at that wave. In this example, a friendship exists between persons 2 and 4. As this network is undirected, only the bottom triangular portion is stored.
Note. Time for a dyad begins when the relationship starts. For example, "Time = 0" is Wave 3 for pair 1,2 but Wave 2 for pair 1,3. The "event" represents when the friendship ended (if applicable).

interest is a binary indicator y of whether the relationship ends at time t as discussed in the preceding. Thus, time in the model is dyad dependent and may represent different actual waves of data collection for different dyads. While the time scores need not be equidistant in terms of the actual amount of time between subsequent scores, the distance between the same periods for different pairs should be consistent.

Often in multilevel models, the individuals or occasions within individuals are the lowest level units of the analysis. However, the model described here is built in order to understand the relationship's duration, an outcome of the pair of individuals. We thus have pairs nested within individuals (Goldstein, 2011, p. 260). Each pair is measured over time, creating a three-level structure. The model thus aims to understand whether and when the pair's relationship ends, accounting for the dependency that occurs from the fact a pair is nested within two individuals, and these individuals contribute to multiple pairs.

This three-level structure is visualized in Figure 1, where the arrows represent the nesting structure and to which higher-level unit the lower-level unit belongs (Rasbash & Browne, 2008). Here, we have time nested within pairs that are then nested within individuals; for example, friendship between individuals 1 and 2 is recorded by "pair 1,2" over three periods. While pairs are made up of only two individuals, individuals may be involved in numerous friendship pairs.

To formalize the model, let i and i' be two individuals within a pair p, with the total number of individuals I across pairs P. The hazard h of relationship duration, which represents the risk or conditional probability of friendship dissolution given the friendship had not yet dissolved, is modeled as a function of a vector of predictors $\boldsymbol{x}$. These predictors could be time- as well as pair-level covariates, any of which may be allowed to vary over time (e.g., there could be an effect of time, pair's average depression, and an interaction between time and the pair's average depression). These predictors have an associated vector of fixed effects $\boldsymbol{\beta}$. A random effect u is added for each individual within the pair, making the model a multiple-membership multilevel model in order to account for each individual's underlying propensity for their friendships to end (Goldstein, 2011; Rasbash & Browne, 2008). The random effects for all individuals are assumed to be normally distributed with mean 0 and common variance σ^2. As in Equation (2), it is assumed that once the mutual ties are dissolved, the ties remain in this state, although it is possible to extend the model for repeatable events. The equations for the FD model then are

$$\begin{aligned} logit\left(h_{pt}\right) &= \boldsymbol{x'}_{pt}\boldsymbol{\beta} + u_i + u_{i'} \\ y_{pt} &\sim Bernoulli\left(h_{pt}\right) \\ u_i &\sim N\left(0, \sigma^2\right), u_{i'} \sim N\left(0, \sigma^2\right). \end{aligned} \tag{3}$$

A high random effect, relative to a low random effect, would imply that an individual's friendships are more likely to dissolve. Unlike most multiple membership models, there is no need to add different weights for different dyad members to the random effects as each individual is assumed to be an equal member of the dyad.[7]

The FD model in Equation (3) represents the conditional probability of friendships ending at each period, given they had survived until that period. Once this hazard function is modeled, the survival function can be formulated according to the fitted hazard function to understand the likelihood of friendship surviving until a certain period:

$$S_{pt} = \prod_{m=1}^{t}\left(1 - h_{pt}\right). \tag{4}$$

The lifetime distribution probability, in this case the likelihood of a friendship ending by a certain period, can be then estimated by $D_{pt} = 1 - S_{pt}$.

In addition to answering specific research questions that are important in their own right, the modeling framework discussed in this article has an advantage of being easily applied to networks without clearly defined bounds (e.g., changing node set, sample from larger network) and can easily handle censored event times (e.g., missing ties). In addition, model fitting of sparse data that is a

[7] For example, a multiple-membership model with school effects may have a weight that approximates the length of time spent in each school for students who transition between schools.

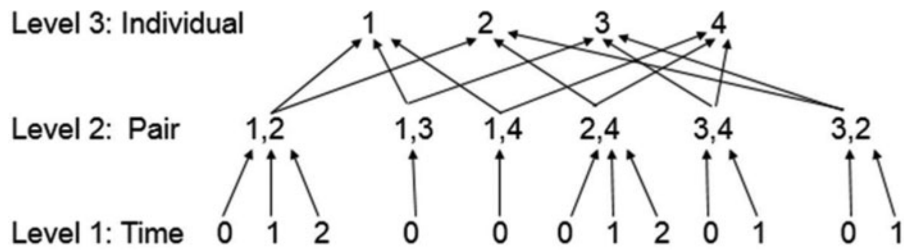

Figure 1. Friendship duration model nesting diagram; multiple membership structure.

ubiquitous problem for social network data—due to most dyads being unconnected versus connected—is mitigated as the focus remains on specific dyadic processes rather than the entire network.

Applied to a pair-period data set, the FD model as specified in the preceding can be fit with standard generalized linear mixed model software as long as the software allows for the estimation of multiple membership effects. A thorough review of estimation options as well as estimation issues related to GLMMs is outside the scope of this article (Tuerlinckx, Rijmen, Verbeke, & De Boeck, 2006; Rodríguez & Goldman, 2001; Zeger & Karim, 1991; Raudenbush & Bryk, 2002). However, the methods here are especially related to estimation of GLMMs with multiple membership structures (Cho & Rabe-Hesketh, 2011; Rasbash & Goldstein, 1994; Rasbash & Browne, 2008). For a comparison of different statistical packages for fitting logistic random-effects models, see Li, Lingsma, Steyerberg, and Lesaffre (2011), who note that different software implementations ranging across both frequentist and Bayesian approaches produce similar results for large data sets but not necessarily for smaller data sets. To understand how well standard estimators perform under realistic scenarios for this model, a simulation study is next utilized to evaluate performance under relevant conditions for the FD model.

Simulation study

The goal of this simulation study is to test the practicality of the proposed model under a set of nonexhaustive but informative conditions. While multilevel event history models are by no means unique, nor are multilevel models with multiple membership structures, these models have yet to be tested together in the types of conditions that would arise in social network data such as in the empirical example in the next section. The simulation was designed to reflect conditions that might be seen in practice, influenced in large part by preliminary analyses of the data discussed later in this article and in a related social network study (Dean, 2015, Chapter 3). The main goals of the simulation are to understand how the hazard rate and sample size—in terms of both the number of individuals and the number of friendships per individual—influence the ability to recover the fixed and random effects of the model. In addition, the simulation study is designed to reveal the influence of the estimation method chosen and the amount of dependence in terms of the magnitude of the random-effect variance.

The hazard function for this simulation study is constant in both the population-generating model and the fitted model (i.e., single intercept), and the number of periods is held constant at five to keep the scope of the simulation reasonable while focusing on more interesting conditions. One binary pair-specific true effect is included in all conditions to test recovery, with a regression coefficient of 0.5 and a balanced mix of the two groups in the population-generating model (e.g., in practice, this could be the effect of "same-gender" versus "different-gender" friendship pair with half of the pairs being one type versus another). One pair-specific null effect is included to test false recovery with the predictor generated from the mean of each individual's predictor coming from an $N(0,1)$ distribution (e.g., in practice, this could be testing for an effect of the mean depression of the pair where no effects exist).

The number of individuals is varied at 100 or 500, reflecting reasonable sample sizes for relatively small and relatively large network studies that are seen in psychology and related disciplines (Schaefer, Light, Fabes, Hanish, & Martin, 2010; Ojanen, Sijtsema, & Rambaran, 2013). The average number of links per node is varied at two and six, again aiming to reflect a reasonable low and high range of ties per node seen in practice (Dijkstra, Cillessen, & Borch, 2013; Ahn & Rodkin, 2014). Both of these types of conditions reflect similar values to what is commonly seen in empirical applications, such as the one that follows in this article. Whether a link is ever to occur between nodes is generated randomly until the average links per node in the network reaches this number; example distributions of the number of links per node are shown in Figure 2. The timing of the link dissolution is then simulated with the logit link function as specified in Equation (3).

The hazard risk is varied between "low" and "high" risk. Low risk is defined specifically as an intercept within

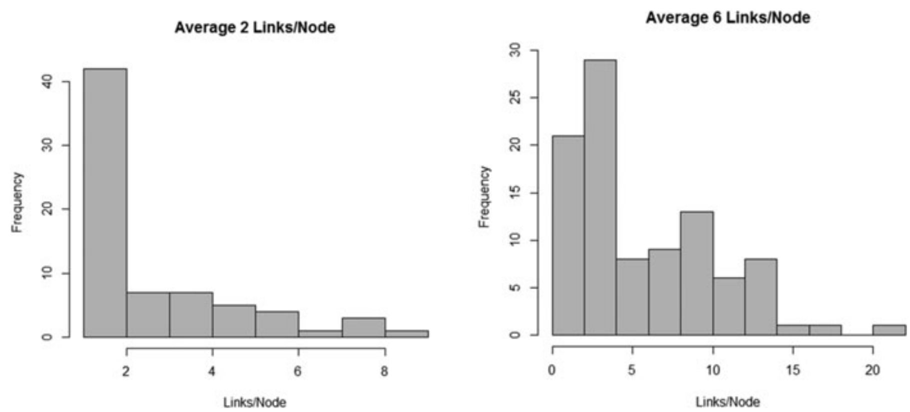

Figure 2. Links-per-node simulation factor: example distributions of links per node across individuals in two data sets, one with an average of two links per node and the other with an average of six links per node, with frequency representing the number of individuals.

the logit function of -3, while high risk is an intercept of 0.05. These values were chosen after preliminary analyses of the empirical applications in this article and a related social network study (Dean, 2015, Chapter 3), as the "high" risk condition reflects a hazard function similar to what the friendship duration data reveal in the next section of this article while the "low" risk condition reflects a hazard function similar to the data analyzed in Chapter 3 of Dean (2015). These parameter choices result in the hazard and survival functions plotted in Figure 3, when random effects are held at the population average. High risk implies a probability of 0.51 of the event occurring at any period for pairs that are eligible to experience the event and only a 0.03 probability of the event not occurring by the last period; low risk implies a conditional probability of 0.05 of the event occurring at any period, with a cumulative probability of 0.78 of the event not occurring by the last period.

The random effect variance is varied at two levels ($\sigma^2 = \{0.35, 1.40\}$). Previous research has often found downward bias in the variance component estimates obtained from GLMMs fitted to binary outcomes, the magnitude of which depends on a number of factors, including the estimation method and the size of the random-effect variance. The smaller random-effect variance was chosen because preliminary analyses of the empirical data in this article indicate a value close to this, without covariates in the model. The higher random-effect variance was chosen to test the model with a stronger dependence structure, but low enough to reflect an effect that might be seen in practice with similar network data. Note that while the standard errors of the random-effect variance are reported in the results, there are limitations to using these standard errors for significance testing in multilevel models, especially for models with nonnested data structures (Schaalje, McBride, & Fellingham, 2002).

We compared two methods of estimation, a frequentist penalized quasi-likelihood (PQL) estimator and a Bayesian Markov-chain Monte Carlo (MCMC) estimator.

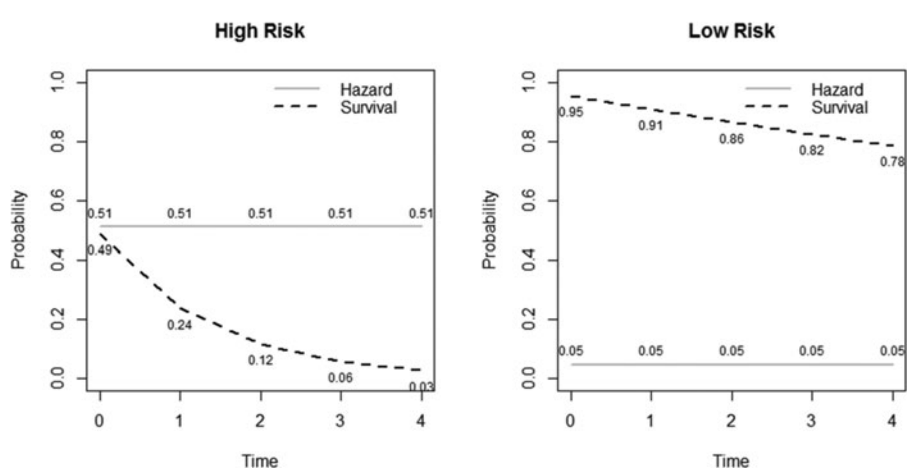

Figure 3. Hazard factor definition. Resulting hazard and survival functions at "high" and "low" risk, keeping random effects at the population average.

Table 2. Friendship duration model: Simulation conditions summary.

Number of nodes	Links per node	Hazard	Random-effect variance	Estimation
100	2	low	0.35	PQL
500	6	high	1.40	MCMC

PQL was implemented in the GLIMMIX procedure of SAS.[8] MCMC techniques were implemented through the MCMCglmm package in R using weak priors (Hadfield, 2010).[9] Previous research has revealed that PQL is computationally efficient and is robust to starting values, performing better with larger cluster sizes and smaller variance components, but has been found to result in fixed- and random-effect estimates biased toward zero when modeling binary outcomes (Rodríguez & Goldman, 1995; Rasbash & Goldstein, 1994; Bauer & Sterba, 2011). MCMC is expected to perform best with a large number of clusters.

In summary, the FD model is tested under five factors, together constituting 32 total simulation conditions (2 × 2 × 2 × 2 × 2), which are summarized in Table 2. The models are replicated 1,000 times between the first four factors, and the same data are then used to fit the model separately by PQL and MCMC.

Results

Performance is assessed by examining bias of the different parameter estimates (Table 3), the standard deviation of the estimates compared to the standard error of the estimates (Table 4), Type I error of the null effect, and power of the pair-level effect (Burton, Altman, Royston, & Holder, 2006). For estimates from MCMC, the estimate is computed as the mean of the posterior distribution, and the standard error is the standard deviation of the posterior distribution. Results are listed in Table 3 and Table 4 and then summarized after the tables by examining specific conditions as well as aggregating across factors to determine how well the model is able to recover the fixed- and random-effect estimates.

Fixed-effects recovery

First examining recovery of the intercept, both MCMC and PQL estimation methods have small absolute bias for the high-hazard condition (when $\beta_0 = 0.05$)[10] and larger absolute bias for the low-hazard condition (when $\beta_0 =$ -3) with MCMC biased away from zero and PQL biased toward zero (see Figure 4). While bias is relatively large in some cases for the low-hazard condition (as large as -0.79 with relative bias of 26%), this condition provides a rather stringent test as the event occurs extremely rarely. Aggregating across estimation methods, there is small or negligible bias in recovering the intercept for other factors. Examining results within estimation method, however, MCMC results in less average absolute bias when increasing from 100 to 500 nodes (0.39 versus 0.23) while the average absolute bias actually increases slightly for PQL estimation when going from 100 to 500 nodes (0.16 versus 0.19). Both MCMC and PQL result in less average absolute bias when increasing from two to six links per node.

Bias in recovery of the true effect (pair-level covariate, $\beta_1 = 0.5$ across all conditions) generally follows a similar pattern. Here, MCMC estimation is again biased away from zero while PQL is biased toward zero, while each other factor reveals little bias when aggregating across the other factors. MCMC tends to result in larger relative bias than PQL (MCMC with average relative bias of 18% versus PQL with average relative bias of 10%), with the difference especially apparent for small sample sizes and with smaller links per node. MCMC estimation again tends to result in less average absolute bias when increasing from 100 to 500 nodes (0.11 to 0.08) while the average bias for PQL between the two levels is similar on average (0.05 for both levels). The difference between a high and low intercept has little effect on bias of recovering the true pair-level covariate effect.

Moving to examine efficiency, and first examining recovery of the intercept, MCMC estimation has larger variability across replications than PQL (for the intercept, average standard deviation across conditions was 0.36 for MCMC versus 0.21 for PQL). The worst estimation of the standard error of the intercept estimates is for MCMC estimation for 100 nodes and two links per node where the standard deviation of the estimates is approximately twice the mean standard error of the estimates; however, the standard deviation of the estimates and the mean standard error of the estimates are nearly identical for other conditions, including all conditions with PQL estimation. Aggregating across estimation methods, there is a clear pattern in variability where the smaller number of nodes

8 Although multiple membership and cross-classified models can be reformulated as multilevel models with nested random effects , the approach requires the evaluation of integrals with high dimensions and is computationally burdensome. Numerical quadrature is not available as an option for multiple membership and cross-classified random effects using GLIMMIX, as "METHOD=QUAD" requires that model can be processed by subjects.

9 Burn-in was set to 50,000 and the number of iterations at 100,000 with a thinning interval of 50 across all replications after trying different values and examining the autocorrelation between successive stored iterations for a few replications. The random effect was specified to have a univariate inverse Wishart prior with the variance at the limit set to 1 (V=1) and varying degree of belief parameter set to 0.002 (nu=0.002) as is commonly used for variance priors. A small set of sensitivity analyses revealed little substantive change in results with minor change in these values.

10 Although the relative bias in the high-hazard condition is often quite large, the absolute bias is small enough that one can argue the model performs reasonably well in this case.

Table 3. Friendship duration model bias and relative bias of parameter estimates.

			$\beta 0$			$\beta 1 = 0.5$			$\sigma 2$			
Estimation	Nodes	Links	TRUE	Bias	Relative bias	Bias	Relative bias	$\beta 2 = 0$ Bias	TRUE	Bias	Relative bias	Prop.*
MCMC	*100*	*2*	*−3*	−0.71	0.24	0.13	**0.27**	0.00	0.35	0.56	**1.61**	NA
MCMC	*100*	*2*	*−3*	−0.79	**0.26**	0.15	**0.29**	0.02	1.4	1.47	**1.05**	NA
MCMC	*100*	*2*	*0.05*	0.05	**1.01**	0.09	0.19	0.00	0.35	0.12	**0.36**	NA
MCMC	*100*	*2*	*0.05*	0.14	**2.71**	0.15	**0.30**	−0.02	1.4	1.05	**0.75**	NA
MCMC	*100*	*6*	*−3*	−0.45	0.15	0.05	0.09	0.01	0.35	0.07	0.20	NA
MCMC	*100*	*6*	*−3*	−0.48	0.16	0.09	0.17	0.00	1.4	0.49	**0.35**	NA
MCMC	*100*	*6*	*0.05*	0.03	**0.66**	0.10	0.20	0.00	0.35	0.17	**0.48**	NA
MCMC	*100*	*6*	*0.05*	0.06	**1.11**	0.09	0.17	0.01	1.4	0.63	**0.45**	NA
MCMC	*500*	*2*	*−3*	−0.43	0.14	0.06	0.11	0.00	0.35	0.05	0.15	NA
MCMC	*500*	*2*	*−3*	−0.47	0.16	0.06	0.13	−0.01	1.4	0.49	**0.35**	NA
MCMC	*500*	*2*	*0.05*	0.02	**0.33**	0.10	0.21	0.00	0.35	0.15	**0.44**	NA
MCMC	*500*	*2*	*0.05*	0.03	**0.54**	0.09	0.17	−0.01	1.4	0.63	**0.45**	NA
MCMC	*500*	*6*	*−3*	−0.43	0.14	0.05	0.10	−0.01	0.35	0.07	0.19	NA
MCMC	*500*	*6*	*−3*	−0.44	0.15	0.07	0.13	0.00	1.4	0.37	**0.27**	NA
MCMC	*500*	*6*	*0.05*	0.01	0.24	0.10	0.20	0.00	0.35	0.15	**0.43**	NA
MCMC	*500*	*6*	*0.05*	0.02	**0.39**	0.09	0.18	0.00	1.4	0.56	**0.40**	NA
PQL	*100*	*2*	*−3*	0.14	−0.05	−0.01	−0.02	0.00	0.35	−0.04	−0.11	0.88
PQL	*100*	*2*	*−3*	0.45	−0.15	−0.08	−0.15	0.02	1.4	−0.62	**−0.44**	1.00
PQL	*100*	*2*	*0.05*	0.00	−0.05	−0.05	−0.10	0.00	0.35	−0.09	**−0.26**	0.97
PQL	*100*	*2*	*0.05*	−0.09	**−1.87**	−0.08	−0.17	−0.01	1.4	−0.59	**−0.42**	1.00
PQL	*100*	*6*	*−3*	0.16	−0.05	−0.02	−0.05	0.01	0.35	−0.05	−0.14	0.99
PQL	*100*	*6*	*−3*	0.37	−0.12	−0.04	−0.08	0.00	1.4	−0.40	**−0.29**	1.00
PQL	*100*	*6*	*0.05*	0.01	0.11	−0.03	−0.06	0.00	0.35	−0.05	−0.14	1.00
PQL	*100*	*6*	*0.05*	−0.04	**−0.87**	−0.07	−0.13	0.01	1.4	−0.40	**−0.28**	1.00
PQL	*500*	*2*	*−3*	0.22	−0.07	−0.02	−0.05	0.00	0.35	−0.10	**−0.29**	1.00
PQL	*500*	*2*	*−3*	0.52	−0.17	−0.10	−0.20	0.00	1.4	−0.69	**−0.49**	1.00
PQL	*500*	*2*	*0.05*	−0.04	**−0.76**	−0.04	−0.08	0.00	0.35	−0.10	**−0.30**	1.00
PQL	*500*	*2*	*0.05*	−0.13	**−2.58**	−0.10	−0.21	0.00	1.4	−0.64	**−0.46**	1.00
PQL	*500*	*6*	*−3*	0.17	−0.06	−0.02	−0.03	0.00	0.35	−0.06	−0.18	1.00
PQL	*500*	*6*	*−3*	0.38	−0.13	−0.05	−0.10	0.00	1.4	−0.43	**−0.31**	1.00
PQL	*500*	*6*	*0.05*	−0.02	**−0.46**	−0.03	−0.06	0.00	0.35	−0.07	−0.20	1.00
PQL	*500*	*6*	*0.05*	−0.06	**−1.29**	−0.06	−0.13	0.00	1.4	−0.41	**−0.29**	1.00

Note. Recovery of the parameters across different conditions (factors italicized), with relative bias greater than 25% bolded.
*Proportion of simulations where the random effect variance parameter was greater than 0. NA implies not applicable, as the prior distribution on the variance component when using MCMC restricts the result to always be greater than 0.

and number of links per node result in greater variability in the estimates of the intercept; a larger random-effect variance also results in greater estimate variability.

Similarly, examining efficiency in recovery of the true effect reveals that MCMC estimation as well as a smaller number of nodes, smaller average links per node, and larger random-effect variance result in greater variability in the estimates. The difference between 100 and 500 nodes makes the largest effect on the power to detect a significant effect (on average 0.39 versus 0.86), although power is higher for larger number of links per node, smaller random effect variance, and the high intercept condition. Despite the differences found in the precedinge, PQL and MCMC result in similar power to detect a significant effect, as well as the same pattern for power across the different factors.

Finally, Type I error of the null effect (pair-level covariate, $\beta_2 = 0$ across all conditions) is as expected, around 0.05 for $\alpha = 0.05$ for all conditions. There is negligible bias in the estimates across all conditions, and similar to recovery of the intercept and true effect, the simulation reveals that MCMC estimation, a smaller number of nodes, smaller average links per node, and larger random effect variance result in greater variability in the estimates, while the intercept factor has little effect.

Random-effects recovery

The random-effect variance is consistently overestimated by MCMC (average relative bias of 50%) and underestimated by PQL (average relative bias of 29%), for both low and high random-effect conditions. The bias is quite severe with a smaller number of nodes and links per node, especially for MCMC; however, we again see an improvement in average absolute bias for MCMC when moving from 100 to 500 nodes (0.66 versus 0.34) but not for PQL (0.26 versus 0.31). The bias in estimating the random-effect variance is consistent with previous research on binary outcome models with random effects, especially considering the number of links per node (equivalent to the number of objects per cluster) is so small in such network data (Rodríguez & Goldman, 2001; Rodríguez & Goldman, 1995).

The standard error of the random-effect variance also tends to be overestimated by MCMC, with the exception

Table 4. Friendship duration model standard error recovery.

			$\beta 0$				$\beta 1 = 0.5$			$\beta 2 = 0$			$\sigma 2$			
Estimation	*Nodes*	*Links*	*TRUE*	SD	M (SE)	M (SE) / SD	SD	M (SE)	M (SE) / SD	SD	M (SE)	M (SE) / SD	*TRUE*	SD	M (SE)	M (SE) / SD
MCMC	*100*	*2*	*−3*	1.08	0.55	**0.51**	0.76	0.54	**0.71**	0.47	0.45	0.95	0.35	7.10	1.37	**0.19**
MCMC	*100*	*2*	*−3*	0.94	0.76	0.81	0.71	0.65	0.92	0.67	0.63	0.94	1.40	4.06	3.04	**0.75**
MCMC	*100*	*2*	*0.05*	0.37	0.38	1.01	0.46	0.44	0.97	0.39	0.37	0.95	0.35	0.43	0.55	**1.27**
MCMC	*100*	*2*	*0.05*	0.59	0.59	1.00	0.61	0.59	0.96	0.56	0.57	1.03	1.40	1.47	1.81	1.23
MCMC	*100*	*6*	*−3*	0.27	0.27	1.00	0.27	0.27	0.99	0.26	0.24	0.94	0.35	0.26	0.31	1.18
MCMC	*100*	*6*	*−3*	0.44	0.41	0.93	0.31	0.30	0.96	0.39	0.38	0.97	1.40	0.66	0.84	**1.28**
MCMC	*100*	*6*	*0.05*	0.25	0.24	0.98	0.23	0.24	1.06	0.25	0.24	0.97	0.35	0.20	0.27	**1.37**
MCMC	*100*	*6*	*0.05*	0.37	0.38	1.04	0.29	0.28	0.96	0.38	0.38	0.99	1.40	0.58	0.77	**1.33**
MCMC	*500*	*2*	*−3*	0.21	0.20	0.93	0.21	0.21	1.00	0.16	0.16	1.01	0.35	0.25	0.28	1.13
MCMC	*500*	*2*	*−3*	0.26	0.27	1.03	0.26	0.25	0.97	0.23	0.23	1.01	1.40	0.53	0.70	**1.33**
MCMC	*500*	*2*	*0.05*	0.17	0.16	0.96	0.19	0.19	1.02	0.16	0.16	0.99	0.35	0.16	0.21	**1.30**
MCMC	*500*	*2*	*0.05*	0.24	0.24	0.99	0.24	0.24	1.00	0.22	0.23	1.04	1.40	0.44	0.57	**1.29**
MCMC	*500*	*6*	*−3*	0.12	0.12	0.99	0.11	0.12	1.07	0.11	0.11	0.97	0.35	0.09	0.12	**1.36**
MCMC	*500*	*6*	*−3*	0.18	0.18	0.98	0.13	0.13	1.00	0.16	0.16	1.01	1.40	0.26	0.33	**1.28**
MCMC	*500*	*6*	*0.05*	0.11	0.11	0.97	0.10	0.11	1.08	0.10	0.11	1.05	0.35	0.09	0.11	1.23
MCMC	*500*	*6*	*0.05*	0.17	0.17	0.99	0.13	0.12	0.95	0.16	0.16	1.02	1.40	0.23	0.32	**1.38**
PQL	*100*	*2*	*−3*	0.35	0.35	1.01	0.41	0.43	1.05	0.34	0.34	1.00	0.35	0.25	0.30	1.21
PQL	*100*	*2*	*−3*	0.40	0.39	0.97	0.44	0.44	1.00	0.39	0.38	0.98	1.40	0.32	0.40	1.24
PQL	*100*	*2*	*0.05*	0.27	0.27	0.98	0.35	0.35	1.00	0.29	0.28	0.97	0.35	0.16	0.18	1.10
PQL	*100*	*2*	*0.05*	0.35	0.35	1.00	0.39	0.40	1.02	0.36	0.36	1.00	1.40	0.30	0.33	1.09
PQL	*100*	*6*	*−3*	0.22	0.22	1.00	0.23	0.24	1.03	0.22	0.21	0.96	0.35	0.14	0.14	1.00
PQL	*100*	*6*	*−3*	0.31	0.29	0.94	0.24	0.25	1.02	0.29	0.28	0.98	1.40	0.27	0.29	1.06
PQL	*100*	*6*	*0.05*	0.17	0.17	0.98	0.15	0.16	1.06	0.17	0.17	0.99	0.35	0.09	0.09	1.00
PQL	*100*	*6*	*0.05*	0.26	0.27	1.05	0.21	0.22	1.04	0.28	0.27	0.97	1.40	0.25	0.26	1.03
PQL	*500*	*2*	*−3*	0.15	0.15	0.98	0.18	0.18	1.01	0.14	0.14	0.99	0.35	0.10	0.11	1.12
PQL	*500*	*2*	*−3*	0.16	0.17	1.03	0.18	0.19	1.04	0.16	0.16	1.01	1.40	0.13	0.16	1.23
PQL	*500*	*2*	*0.05*	0.12	0.12	0.97	0.15	0.15	1.01	0.12	0.12	1.00	0.35	0.07	0.07	1.02
PQL	*500*	*2*	*0.05*	0.15	0.15	1.01	0.17	0.17	1.01	0.15	0.15	1.02	1.40	0.12	0.14	1.14
PQL	*500*	*6*	*−3*	0.10	0.10	0.98	0.10	0.10	1.04	0.09	0.09	1.02	0.35	0.06	0.06	0.99
PQL	*500*	*6*	*−3*	0.13	0.13	1.00	0.11	0.11	0.98	0.12	0.12	1.04	1.40	0.11	0.12	1.13
PQL	*500*	*6*	*0.05*	0.08	0.08	1.01	0.08	0.09	1.08	0.08	0.08	1.03	0.35	0.05	0.05	0.90
PQL	*500*	*6*	*0.05*	0.12	0.12	1.01	0.09	0.10	1.07	0.12	0.12	1.00	1.40	0.10	0.11	1.13

Note. Standard deviation of parameter estimates compared to mean standard error across replication (factors italicized) with conditions bolded when the ratio is off by more than 25%.

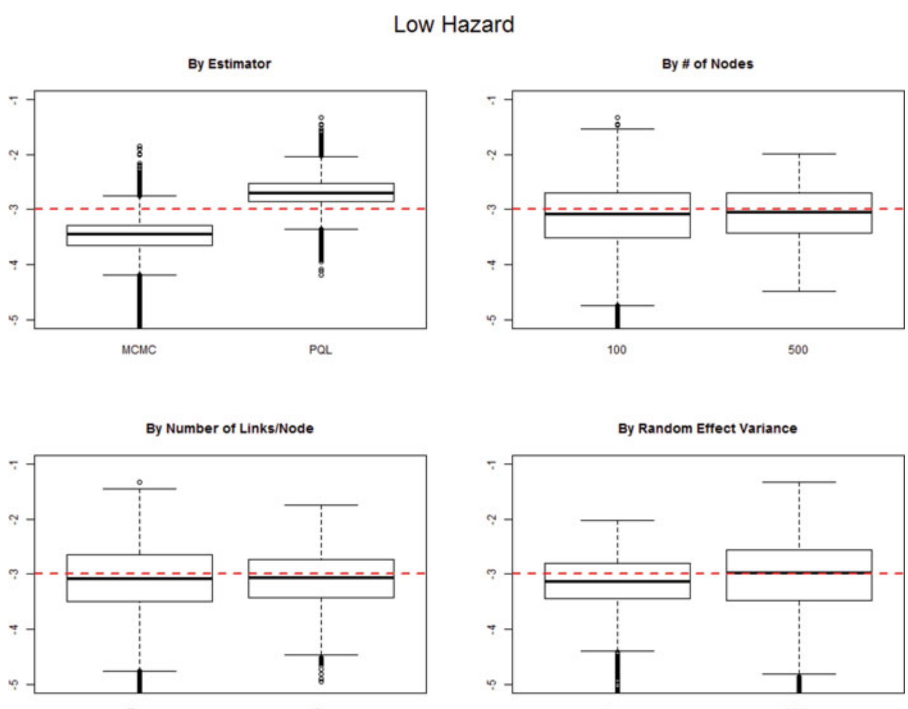

Figure 4. Intercept recovery, low hazard condition. Boxplot of parameter estimates across simulations, aggregated by each factor. The red dotted line indicates the true parameter value.

of small number of nodes and links per node and low hazard when the standard error is largely underestimated (Table 4). There is less variability in the estimates when the number of nodes, as well as the number of links per node, is higher and when the intercept is higher (higher risk of event occurrence).

Summary

In summary, although recovery of the random effect tends to be poor, the model recovers the fixed-effect parameter estimates relatively well, especially when the number of nodes and number of links per node are larger. Increasing the number of nodes has the largest effect on the power to detect a significant pair-level effect and improves the bias of MCMC estimation results although not PQL estimation results. An increase in the number of links per node also has a large influence on power and improves results in terms of both bias and variability for both MCMC and PQL estimation, as does a smaller random-effect variance. Finally, PQL estimation tends to have less variability in the conditions of this study than MCMC, and recovery tends to be better for the high intercept condition when the risk of event occurrence is higher. Now that the model has been tested under relevant conditions to real social network data, we next apply the FD model to high school friendship duration data, the empirical example that motivated the development of this model.

Application to high school network data

In this empirical example, the processes leading to high school friendship dissolutions and the length of high school friendships are examined, especially in relation to the influence of gender and depression. Students were recruited at the beginning of high school, in the fall of their ninth grade. A total of 423 students were recruited from three different schools in North Carolina. All ninth graders within the three schools had the opportunity to consent. Data collection began in February of ninth grade (in 2009), and collections took place at approximately 6-month intervals through the spring of their twelfth-grade year, with seven waves of data collection total.[11]

At each wave of data collection, students were given a current roster and allowed to nominate an unlimited number of individuals that they considered to be their friends. A friendship pair was considered to exist when both individuals within the pair nominated each other as friends. A total of 814 friendship pairs existed for at least two periods of which 194 were male friendship pairs, 391 were female friendship pairs, and 229 were mixed-gender

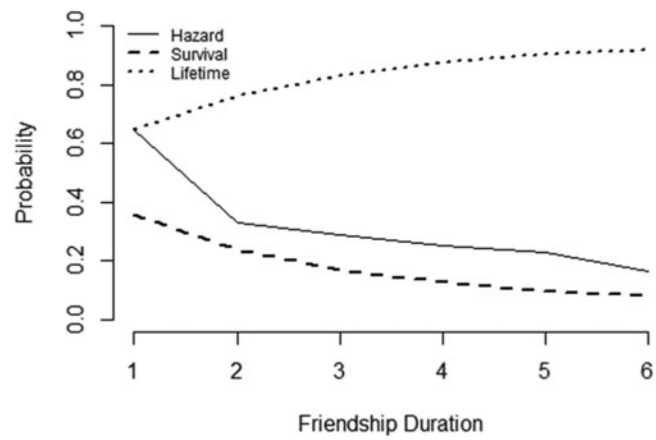

Figure 5. Sample estimated hazard, survival, and lifetime distribution functions for high school friendship data without a formal statistical model.

friendship pairs. The friendship pairs were drawn from a total of 339 individuals (42% male). For each pair, time was coded to indicate when the friendship began (rather than the wave of data collection), and a binary indicator of friendship ending was created if either or both of the individuals failed to nominate the other individual within the pair as a friend. Friendship pairs were included for all periods when both individuals within the pair provided nominations until the first friendship dissolution or censoring occurred.

The sample observed hazard, survival, and lifetime distribution functions are plotted in Figure 5, without modeling or accounting for the multiple-membership dependence structure. The hazard function indicates friendships are most likely to end at the first period after the friendship began, but if the friendship continues to exist, it becomes less likely to end over time. The sample observed functions also indicate that high school friendships in this sample are highly fickle, in that 64% end after the first period and only 8% are estimated to survive from the start of high school until the end.

The FD model is applied to longitudinal friendship nomination data using PQL through SAS Glimmix, as the simulation study revealed a tendency for PQL to have less variable parameter estimates albeit similar absolute bias for fixed- and random-effect recovery. Gender and depression are entered into the model as the main predictors of friendship dissolution. Specifically, type of pair (males, females, mixed) is entered as two dummy-coded variables with mixed as the reference group, and depression is investigated in terms of both the average depression of the pair and absolute difference in depression of the pair. Depression is measured at each period through a mood and feelings questionnaire (MFQ) (Costello & Angold, 1988).

[11] Although there are seven waves of collection, there are only six periods for purposes of the data structure as friendship cannot start and end at the same time period with this discrete-time collection.

Table 5. Friendship duration model application: Fixed-effects results from applying the friendship duration model to the high school friendship data.

Effect	Estimate	Standard error	DF	*t* value	Pr > |t|
Fixed effects					
Intercept	**0.931**	**0.218**	**513.1**	**4.27**	**< 0.001**
Time	**−0.484**	**0.063**	**1286**	**−7.70**	**< 0.001**
Male pair (mixed gender reference)	−0.284	0.188	488.2	−1.51	0.133
Female pair (mixed gender reference)	**−0.508**	**0.160**	**407.7**	**−3.18**	**0.002**
Pair mean depression	−0.432	0.324	442.9	01.33	0.183
Absolute difference depression	**0.607**	**0.258**	**1286**	**2.35**	**0.019**
Mutual friendships	**−0.263**	**0.072**	**357.1**	**−3.66**	**< 0.001**
Nonmutual friendships	**0.130**	**0.027**	**429**	**4.82**	**< 0.001**
Variance/covariance parameters					
Multiple membership effect	0.034	0.046			

Note. Significant effects at $\alpha = .05$ are in bold.

As each individual has a time-varying depression score, and there are two individuals within a pair, the effect of depression can be investigated multiple ways. Here, the average depression of the pair across periods and the average absolute difference in depression in the pair across periods are investigated as predictors within the model.[12] To examine higher-level network effects on dissolution, effects were included for the number of mutual friendships and the number of nonmutual friendships outside the dyad. A mutual friendship for a given pair is defined as occurring when both individuals within the dyad nominate another person as a friend, and a nonmutual friendship is defined as occurring when one person but not both individuals within the pair nominate another person outside the dyad as a friend.

Results

As expected from the sample observed hazard functions, the model finds a high risk of dissolution, yet a decreasing risk of dissolution over time. Thus, high school friendships in this sample are very fickle, yet if they continue to last, they are less likely to end over time. For a mixed-gender pair,[13] the probability of dissolution by the first period after the friendship begins is 0.681, and although the risk of dissolving decreases over time, the cumulative probability of remaining friends through six periods is only 0.04. The fixed-effect estimates and the multiple-membership random-effect variance estimate are displayed in Table 5.

First examining the effect of depression, the average absolute difference in the two individuals' depression is found to have a positive relationship with the likelihood of dissolution, $F(1, 1{,}286) = 5.52$, $p = .02$. This is visualized in Figure 6, panel (A), for female pairs with other covariates held at the average value across all time and pairs; a low value is representing when the pair has the same depression score over time and a high value is representing when the pair is in the 75th percentile for the average absolute difference in depression over time. Thus, pairs who have a larger absolute difference in depression across periods are more likely to end friendships across time than pairs who are more similar in depression. However, no significant effect is found for the pair's average depression on the likelihood of the friendship dissolving. Thus, it was not the pair's overall amount of depression over time that was found to relate to dissolution, but rather how different the pair's depression levels were from one another.

Holding depression constant, gender is found to have an influence on the likelihood of dissolution, $F(2, 195.7) = 5.08$, $p < .01$. Mixed-gender pairs have the highest likelihood of dissolution, while female pairs are the least likely to dissolve friendships over time, as visualized in Figure 6, panel (B), where other covariates are held at the average value across all time and pairs. The difference between the rate of dissolution for mixed-gender and female pairs is significant, $F(1, 407.7) = 10.11$, $p < .01$. In particular, mixed-gender pairs have 1.66 times the odds of high school friendships ending at each period than do female pairs, holding the random effects at the population average. Although mixed-gender pairs are found to have a higher likelihood of dissolution across periods than male pairs as well, no significant difference is found between mixed-gender pairs and male pairs, $F(1, 488.2) = 2.28$, $p = .13$, $OR = 1.33$. In addition, no statistically significant difference between male and female pairs is found in the likelihood of dissolution, $F(1, 100.5) = 1.81$, $p = .18$, although males are found to have 1.25 times the odds of females to dissolve, holding the random effects at the population average.

[12] Each pair has a mean depression score at each period and an absolute difference in depression at each period, and there is variability both between pairs and within pairs. However, using pair-mean centering to investigate the within- and between-pair effects resulted in unstable estimates due to the limited information available in the data to separate these effects. In addition, the results were substantially similar to those presented here when aggregating across time.

[13] This is assuming average value for the covariates across all time and pairs, which is 0.33 for average depression, 0.31 for average absolute difference in depression, 0.62 for mutual friends, and 3.31 for nonmutual friends.

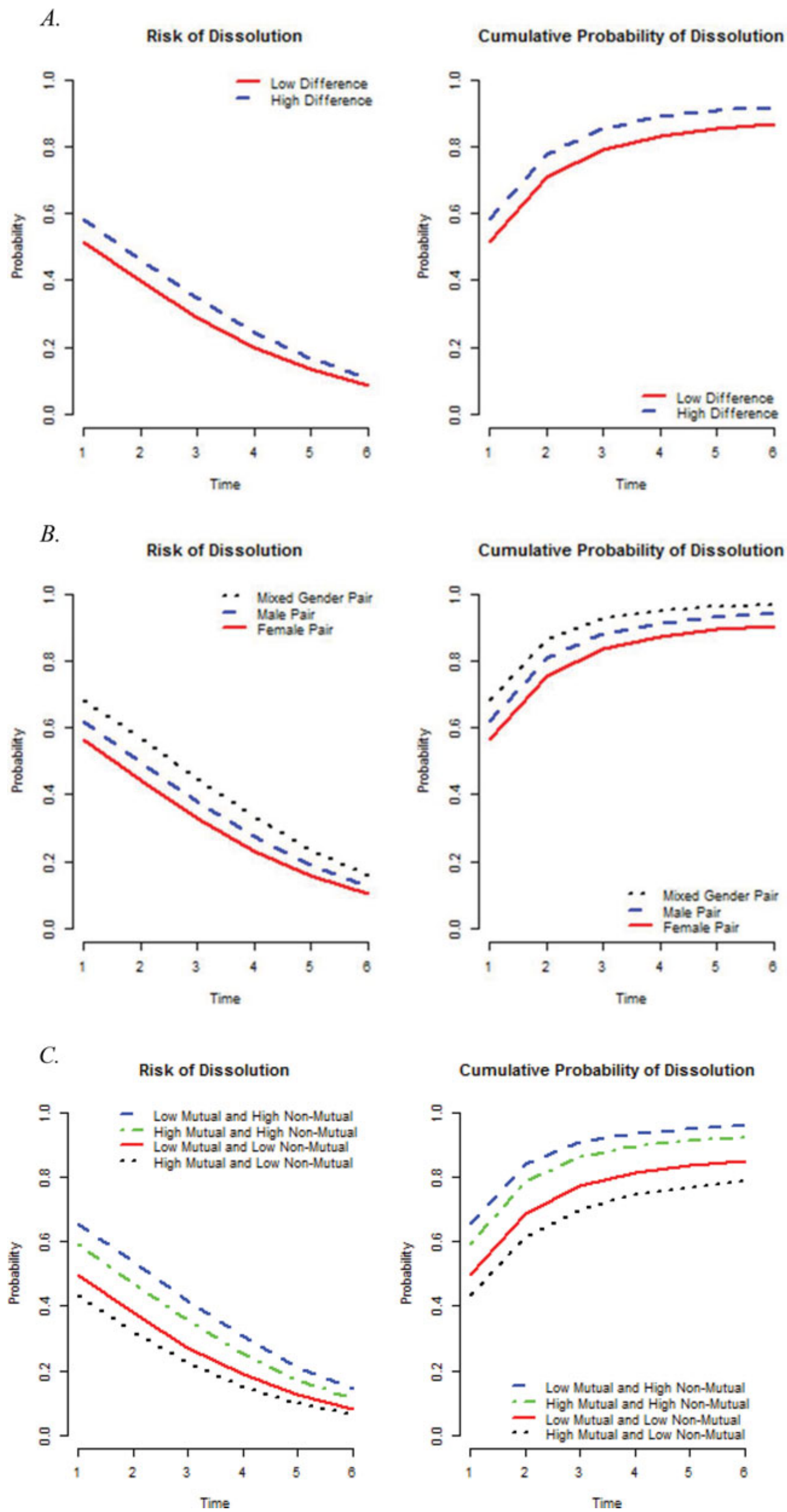

Figure 6. Predicted friendship dissolution: (A) by average absolute difference in depression (pairs with high absolute difference in depression across time periods are more likely to dissolve friendships than pairs who are similar in depression scores); (B) by gender (mixed-gender pairs are the most likely to dissolve across time periods; female pairs are the least likely to dissolve); (C) by number of mutual and non-mutual friends (pairs with low number of mutual friends and high levels of nonmutual friends are most likely to dissolve friendships over time).

The network effect for mutual friendships revealed that as the number of mutual friendships increased, the likelihood of friendship dissolution decreased, $F(1, 357.1) = 13.39$, $p < .01$. Conversely, as the number of nonmutual friendships increased, the likelihood of friendship dissolution increased, $F(1, 429) = 23.23$, $p < .01$. These network effects are visualized in Figure 6, panel (C), where a low number represents none of that type of friend while a high number is representing the 75th percentile across pairs. For example, a pair with no mutual friends but a high number of nonmutual friends is found to be very likely to dissolve their friendship as compared to a pair with a high number of mutual friends and no nonmutual friends (cumulative probability of dissolution of 0.96 by the last period versus cumulative probability of 0.78).

Higher-level interactions between gender and time, gender and depression, and depression and time were tested but not found to have significant effects. Accounting for the effect of depression, gender, mutual and nonmutual friendships in the model, the estimate of the random-effect variance was 0.034 with a standard error of 0.046, suggesting these is little dependence between the dissolution times of friend pairs.[14] Given the findings of the simulation study, however, it is likely that the random effect is underestimated here and is nonetheless important in order to get unbiased estimates of the parameters (Guo & Zhao, 2000).

Conclusion

Individuals are embedded in a complex network of relationships with others around them. While social scientists are interested in studying various aspects of individual nature and society, individuals are often studied in isolation from one another. Standard statistical techniques assume that individuals are randomly sampled from a population, and typically an individual's outcomes are modeled as a function of his or her own characteristics, with the assumption that the behavior of one individual in the sample does not influence another. Yet by focusing on the relationships between individuals and explicitly recognizing the interdependencies between individuals inherent in everyday life, a social network perspective can bring new insight into human nature and processes that influence behavior (Abbot, 1997; White, Boorman, & Breiger, 1976). Social network analysis has given us insight into the effects of communication network structures on a group's ability to complete a task, how position in a social network can influence access to resources, and the influence of urbanism on psychological well-being, for example (Borgatti, Mehra, Brass, & Labianca, 2009).

Although various models have been developed to study how network structures arise, many fundamental questions surrounding the processes leading to specific types of events between dyads within a network remain stubbornly difficult to address. To address questions surrounding friendship dissolution such as how depression influences friendship dissolution over time and accounting for the fact that friendships are embedded within a larger network of relationships, a discrete-time multilevel event history model is utilized building upon the work of de Nooy (2011). This FD model contains a multiple-membership random-effect structure to account for the dependencies that arise from individuals being members of multiple friendships.

The processes leading to specific types of network events can be very different from more general tie-formation processes, which can be modeled in longitudinal social network social-selection process models. Consider as another example that one was interested in the tendencies for individuals to reciprocate friendships within a high school social network. A general-purpose social-selection model may reveal the effect of gender on the likelihood of an edge forming between two individuals within a network, controlling for the effect of reciprocity. For example, there could be a positive reciprocity effect, revealing that individuals are more likely to consider an alter to be a friend if the alter already considered them a friend; controlling for this effect, females may be more likely than males to consider other individuals to be a friend. Individuals are assumed to have the same underlying tendencies to reciprocate, and the covariates are predicting tie formation generally rather than reciprocity.[15] By contrast, the proposed modeling framework allows one to study the effect of gender on the actual outcome of interest—the likelihood of reciprocating. It allows individuals to vary in their underlying tendencies to reciprocate and be the product of reciprocation; the modeling framework also answers questions about the timing of reciprocity, such as the median time to reciprocation, in a straightforward manner as it actually models the risk of reciprocating over time for each dyad.

In addition to answering specific research questions, the model considered in this article has the advantage of being easily applied to networks without clearly defined

[14] To test whether the dependence really is negligible, a model was fit with the covariance parameter fixed at 0. Indeed, the results are substantively similar and would not affect any of the conclusions in this article.

[15] This may be relaxed somewhat by including an interaction term between gender and reciprocity, to understand how gender interacts with reciprocity tendencies to influence the formation of an edge. For example, one may find that females could have an even stronger reciprocity tendency than males. Then to understand whether females may reciprocate at different rates than males, higher-order interactions would be needed. Note that within this standard framework, the timing of reciprocity is not a straightforward product of the model.

boundaries and a changing node set, as the formulation simply requires pair-level data that may change at any wave of data collection (although having more data on the network structure can lead to more informative network covariates). For example, relationship events within the social network Twitter could be analyzed easily by focusing on the dyad over time, accounting for new members and attrition through the standard approach for handling censoring with an event history model. As the modeling framework focuses on specific events between individuals within the network, the ubiquitous sparseness problem of social network data is also not present here, making modeling more straightforward. The simulation study and empirical example are designed to give more insight into the potential advantages and limitations of the modeling framework.

Within the simulation study, the fixed effects were generally recovered reasonably well, except that PQL resulted in estimates biased toward zero while MCMC resulted in estimates biased away from zero. The results suggest that when designing studies to investigate friendship dissolution or network events within dyads using this framework, recovery will be improved if data can be collected from a larger number of individuals. This is because as the number of individuals increased in the simulation study, the bias, variability, and power to detect a true pair-level effect were improved; this was especially true for MCMC, which revealed smaller average absolute bias for the different parameters for 500 nodes than 100 nodes as compared to PQL, which was similar on average between the two levels. PQL is not a consistent estimator, and these results are consistent with prior research showing that bias does not improve (and may even worsen slightly) with more clusters (Bauer & Sterba, 2011). Nevertheless, when data are only available on a smaller number of individuals, PQL produces the most accurate results of the two estimators considered here and should thus be selected.

The results also suggest to researchers that collecting data on more friendships per person can improve results, as a larger number of links per node improved both bias and variability for MCMC and PQL estimation methods and improved the power to detect a significant pair-level effect. Although likely out of researchers' control in the design phase, the simulation study also revealed that recovery was better for both fixed and random effects when the random-effect variance was smaller. Recovery of the random-effect variance was poor for both MCMC and PQL estimation, consistent with prior research on binary outcomes, especially given the number of links per node was so small in the kind of social network data investigated in the simulation study (Rodríguez & Goldman, 1995). While estimation options were not a main focus of this article, it is clear that future research should further investigate this in more detail, given the nature of social network data often leading to very different data structures than those typically studied with multiple-membership multilevel models.

To illustrate the usefulness of the FD model, we demonstrated how it could be applied to investigate processes leading to friendship dissolution and the timing of dissolution over time. The model revealed high school friendships to be very fickle, with a majority ending after one semester; but if the friendship continues, the likelihood of dissolution decreases over time. This is similar to previous research on relationships beginning in middle school (Hartl, Laursen, & Cillessen, 2015). Both gender and depression were found to have a significant influence on dissolution. Mixed-gender pairs had the highest likelihood of dissolving, and female pairs had the lowest likelihood; pairs with similar depression showed a decreased likelihood of dissolution. By contrast, the overall mean level of depression of the individuals did not influence the likelihood of dissolution, an interesting finding as other emotional and behavioral problems have been found to interfere with friendship maintenance (Piehler & Dishion, 2007). Although the dependence structure found in the empirical example after accounting for gender and depression effects was relatively weak, it was only by fitting the FD model that we were able to obtain this information. Other applications may evince higher levels of dependence. The FD model permits one to estimate the degree of dependence in the data and to account for it when examining processes leading to friendship dissolution.

Limitations and extensions

In this article, there was limited inclusion of network effects beyond the dyad in the simulation study, although network effects associated with mutual and nonmutual friendships were considered in the empirical analysis of friendship dissolution. It is possible that higher-level effects could affect the ability to recover lower-level effects, such as the effect of depression on the pair. For example, as was seen in the empirical example, high school friendships may be affected by the friendships connected to the dyad; even the attributes of surrounding friends not within the dyad may have an effect and are another aspect of the network that may be considered in relation to the effects on friendship dissolution. Future work should investigate in more detail the effect of these higher-level characteristics of the network and the robustness of the model results when considering such effects. For example, higher-level friendship groups may be investigated; some possibilities could be to include friendship groups that are predefined as an additional group-level random effect to the model (if such data were available), or potentially one could impose a spatial structure on the individual random effects, where these would be correlated on the basis of some measure of proximity, rather than assuming them to be independent as is done in this article. However, there are many practical difficulties and complexities in deciding how to examine higher-level friendship groups, such as how to decide which dyads are in which groups and the amount of overlap between groups, which are beyond the scope of this article and which deserve careful research. These types of approaches should be studied in more detail and would be important extensions to the modeling framework proposed in this article.

Related to understanding what influences friendship dissolution, it is important to note that within this article, friendship between individuals was defined as specifically occurring when both individuals nominated each other as a friend, and friendship ended when that mutual nomination no longer occurred. Missing data that occurred—such that status of the friendship was unknown as a result of last wave of data collection or dropout of a student from the school, for example—was easily accounted for as censoring in the event history model formulation. By defining friendship in this way, the network is undirected and the effects that are examined are a result of the attributes of both individuals (e.g., pair's mean depression score). Substantively, this formulation is examining the friendship pair as one entity rather than through the lens of individuals. An interesting comparison could be made by instead formulating the problem as "perceived friendship," where friendship is allowed to be directional. This would lead to a different theoretical perspective on the processes leading to dissolution and could reveal different insight into friendship dissolution. For example, an effect could be included for the depression of the person who perceives a friendship and for the person who is the recipient of the perceived friendship. To model perceived friendship dissolution, a cross-classified random-effect structure would be appropriate similar to the model by de Nooy (2011). Future research should compare the substantive conclusions from formulating the processes leading to network effects in different ways. Another avenue of investigation concerns the effect of time (actual wave of data collection rather than time of friendship). Timing effects could be investigated to understand, for example, whether later-forming friendships in high school are more stable than those formed at early stages, by including additional time variables as covariates in the model (e.g., year friendship was established).

A limitation of the model presented in this article is that any potential measurement error in recording friendships was not investigated or accounted for in the model, which has the potential to affect our ability to understand the processes leading to the friendship events of interest. Measurement error would likely decrease our effectiveness in recovering the true effects; in particular, the rate of dissolution may be overestimated in the example in this article as both friends have to agree that they are friends, thus introducing bias when measurement error occurs. Future research should investigate how errors in recording friendships may influence results. It is also important to note that assessing goodness of fit is not straightforward for multilevel models with binary outcomes and is an area of active research (Perera, Sooriyarachchi, & Wickramasuriya, 2016); this is especially true with nonnested data structures, and an extension to this work could be to research potential methods to appropriately assess goodness of fit. The FD model could also be extended in future work to examine repeated events, such as pairs who become friends again after dissolution. This can be done by modeling each friendship spell (Willett & Singer, 1995). For example, by defining each friendship spell separately and adding an indicator in the data set for the spell, an additional random effect can be introduced to account for the dependence introduced by pairs being represented in the data set for multiple distinct spells (Masyn, 2003). Additional considerations should be taken to understand whether there are effects such as an increased likelihood to dissolve friendships as the number of friendship spells for the pair increases.

While there are several possible ways to extend the framework presented here, the FD model can now serve as a baseline upon which other researchers can build in order to investigate the processes leading to friendship dissolution in more detail. With the broader framework of multilevel event history analysis, researchers can directly model an event between individuals embedded within a network, answering questions about the processes leading to the event as well as the timing of that event.

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